

CFT Duals for Ultra-Spinning Gauged Supergravity Black Holes

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Abstract

Recently a new class of asymptotically AdS ultra-spinning black holes has been constructed with non-compact horizon of finite area [1], in which the asymptotic rotation is effectively boosted to the speed of light. We employ this technique to the 4-dimensional $U(1)^4$ and 5-dimensional $U(1)^3$ gauged supergravity black holes. The obtained new exact black hole solutions for both cases possess a non-compact horizon; their topologies are sphere with two punctures. We then demonstrate that the ultra-spinning limit commutes with the extremality condition as well as the near horizon limit for both black holes. We also show that the near horizon extremal geometries of the resulting ultra-spinning gauged supergravity black holes lead to the well-known result which contains an AdS_2 throat. We then obtain the $[(d-1)/2]$ central charges of the dual CFTs. By assuming the Cardy formula, we show that despite the non-compactness of the horizon, microscopic entropy of the dual CFT is precisely equivalent to the Bekenstein-Hawking entropy.

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1 Introduction

The first solution of Einstein field equations which is its only 4d spherically symmetric vacuum solution, describing the first black hole was given by the Schwarzschild metric. Moreover many other black hole solutions have been found until now. Black holes possess a special null surface called event horizon generated by a null Killing vector field, where no object behind it can escape to infinity. Bekenstein discovered that to prevent the violation of the second law of thermodynamics in the presence of a black hole, it must be viewed as a thermodynamic object with entropy. Hawking showed that the black hole acts as a black body with the finite temperature which can radiate away its mass. Thermodynamic behaviour of black holes points to the existence of an underlying black hole microstates structure. One of the main curiosities is: How can these microstates from statistical viewpoints explain the macroscopic Bekenstein-Hawking entropy? However a complete answer has not been given yet, but in string theory, for a large class of extremal supersymmetric black holes this question has been answered [2] that Bekenstein-Hawking entropy can be reproduced from a statistical entropy as the logarithm of the degeneracy of BPS states indeed [3]. Recently a new proposal has been demonstrated in [4] and evaluated in [5] that the microstates of 3-dimensional BTZ black holes can be identified by the conserved charges related to non-trivial diffeomorphisms on the near horizon region.

The Kerr/CFT correspondence [6] provides a rich setup, supporting whatever the states of quantum gravity in the near horizon region of an extremal Kerr-AdS black hole, they are holographically dual to quantum states of a two-dimensional chiral (left-moving part) CFT. Since then many further examples [7, 8] have been investigated for a large class of black hole solutions. In all cases the statistical microscopic entropy of the dual CFT using the Cardy formula [9] precisely agrees with the Bekenstein-Hawking entropy of the black holes.

There is a classification of black hole solutions based on their horizon topology. In particular, the famous Hawking theorem for stationary, asymptotically flat vacuum Einstein black hole solutions in 4-dimension asserts that their horizon topologies necessarily are S^2 . By relaxing some of the Hawking's theorem assumptions, one may come up with some classes of black hole solutions with different horizon topology, such as S^3 and $S^2 \times S^1$ (black ring solutions) horizon topologies in five-dimension. Also in asymptotically AdS spacetimes, the horizon of a black hole may have a compact Riemann surface of any genus g instead of spherical horizons [10], as well as black ring solutions with horizon topology $S^1 \times S^{d-3}$. In the case of adding rotation to them, the horizon would then be non-compact and the obtaining geometries describe a rotating black membranes with horizon topology $\mathbb{H} \times S^{d-4}$. It was shown in four-dimension in [11] and elaborated upon in [12] that in Einstein-Maxwell-Lambda theory or more generally, in the presence of a scalar potential for $\mathcal{N} = 2$ gauged supergravity, one can obtain black hole solutions with non-compact horizon topology with finite area, which can be topologically viewed as spheres with two punctures. Other examples of this kind of topology were constructed in [1, 13] by performing ultra-spinning (super-entropic) limit to Kerr-AdS and d -dimensional multi-spinning Myers-Perry black holes.

The first effort to study ultra-spinning black holes was done by Emparan and Myers [14] for exploring the stability of Myers-Perry black holes in any arbitrarily large angular momentum with keeping mass fixed, in which the rotation parameter $a \rightarrow \infty$. One result is that area decreases as angular momentum increases, and in higher dimensional d , area shrinks to zero. By capturing the null geodesics in the plane of rotation one can realize that the horizon of this kind of ultra-spinning black hole is highly spread out in the plane of rotation while shrinking in the perpendicular direction. Also it was argued that a Gregory-Laflamme-type instability can be seen for these solutions. The analogue limit can be used in the case of rotating AdS black holes in $d \geq 6$ by taking the rotation parameter a , approaches the AdS radius ℓ , which gives the geometry of a black membrane by keeping the mass of the black hole fixed [15]. In [16] another technique is proposed by which one can also perform $a \rightarrow \infty$ while restricting the ratio a/l to remain fixed. Furthermore, Caldarelli et al. have constructed a new type of solution with horizon topology $\mathbb{H}^2 \times S^{d-4}$ by keeping the horizon radius r_+ fixed while zooming in to the pole and taking $a \rightarrow \ell$, which is called hyperboloid membrane limit [15, 17]. Recently a simple ultra-spinning (super-entropic) limit was introduced in [1, 13] for Kerr-Newman-AdS and d -dimensional multi-spinning Myers-Perry black holes. This technique begins with the Kerr-AdS black hole in asymptotically rotating frame, and then boosting this rotation to the maximum value $a \rightarrow l$ while keeping the metric finite by introducing a change to corresponding azimuthal coordinate and finally one can compactify this new coordinate to generate a new rapid black hole solution. The resulting black hole has an interesting feature, which is having a non-compact horizon but finite area. Also it was shown in the context of an extended thermodynamic phase space where the cosmological constant can vary [18], the entropy of some of these new solutions violate the 'Reverse Isoperimetric Inequality', so such black holes are called super-entropic [1, 13].¹

The aim of this work is to find new rotating black hole solutions by taking the recent ultra-spinning (super-entropic) limit [1] on the some existent black holes. The interesting geometry of resulting ultra-spinning black holes motivated us to further explore the applicability of this limit for some gauged supergravity black holes in 4 and 5 dimensions to generate a new sort of black hole solutions. Since Supergravity solutions correspond to consistent string theory backgrounds, microscopic degeneracy of states can be investigated using the AdS/CFT correspondence providing

¹It was shown in [19] for a black hole of given thermodynamic volume the entropy inside a horizon is saturated for (charged) Schwarzschild-AdS black hole. If the entropy of a black hole exceeds its expected maximal entropy it will be denoted as 'super-entropic'.

more motivation for their study in ultra-spinning limit. We will focus specifically on two gauged supergravity solutions in $\mathcal{N} = 2$ and $\mathcal{N} = 4$. After that the Kerr/CFT correspondence for obtaining gauged supergravity ultra-spinning black holes are investigated. It was shown in [20] that despite the non-compactness of event horizons there exists this correspondence for the ultra-spinning Kerr-Newmann-AdS black hole as well as the ultra-spinning limit of minimal gauged supergravity black holes in five dimension.

This paper is organized as follows. In section 2, we consider $U(1)^4$ gauged supergravity black hole in 4-dimension and we shall discuss the geometry of obtained ultra-spinning limit version. We then explore extremality conditions and near horizon limit under ultra-spinning limit for this black hole, indicating that both of them commute with the ultra-spinning limit. Next we provide a brief review of Kerr/CFT correspondence, followed by finding microscopic entropy via Cardy formula which exactly equals to the black hole entropy. In section 3 we have presented a similar analysis for multi-spinning $U(1)^3$ gauged supergravity black hole in 5-dimension. Our conclusions with some remarks are given in Section 4.

2 4-dimensional $U(1)^4$ gauged supergravity with pairwise equal charge

Here, we consider four-dimensional rotating gauged supergravity black hole with pairwise-equal charged which is constructed first in [21] as a $U(1) \times U(1)$ abelian subgroup of the $SO(4)$ gauged $\mathcal{N} = 4$ supergravity. This solution can be consistently embedded in $\mathcal{N} = 8$ gauged supergravity, in which two independent electromagnetic charges can be carried by fields in $U(1)$ subgroups of the two $SU(2)$ sectors in $SO(4) \sim SU(2) \times SU(2)$. Its relevant bosonic Lagrangian is given by

$$\begin{aligned} \mathcal{L}_4 = & R * \mathbb{1} - \frac{1}{2} * d\varphi \wedge d\varphi - \frac{1}{2} e^{2\varphi} * d\chi \wedge d\chi - \frac{1}{2} e^{-\varphi} * F_{(2)2} \wedge F_{(2)2} - \frac{1}{2} \chi F_{(2)2} \wedge F_{(2)2} \\ & - \frac{1}{2(1 + \chi^2 e^{2\varphi})} (e^\varphi * F_{(2)1} \wedge F_{(2)1} - e^{2\varphi} \chi F_{(2)1} \wedge F_{(2)1}) \\ & - g^2 (4 + 2 \cosh \varphi + e^\varphi \chi^2) * \mathbb{1}, \end{aligned} \quad (2.1)$$

where φ is dilaton and χ is axion.¹ Indeed this special solution is a $U(1)^2$ subset of $U(1)^4$ which is arisen by setting the two electric charges equal ($\delta_2 = \delta_4$), and taking the two magnetic charges equal ($\delta_1 = \delta_3$) as well. For more details we refer the reader to [21].

Four field strengths can be written in terms of potentials as

$$F_{(2)1} = dA_{(1)1}, \quad F_{(2)2} = dA_{(1)2}, \quad (2.2)$$

and g denotes the gauge-coupling constant, which is related to the AdS radius ℓ by $g = \ell^{-1}$. Its non-extremal black hole solution in asymptotic rotating frame (ARF) is given by [21]

$$ds^2 = -\frac{\Delta_r}{W} \left(dt - \frac{a \sin^2 \theta}{\Xi} d\phi \right)^2 + W \left(\frac{dr^2}{\Delta_r} + \frac{d\theta^2}{\Delta_\theta} \right) + \frac{\Delta_\theta \sin^2 \theta}{W} \left(a dt - \frac{r_1 r_2 + a^2}{\Xi} d\phi \right)^2, \quad (2.3)$$

¹ In [21] a formalism has been proposed in ungauged $\mathcal{N} = 2$ supergravity coupled to three vector multiplets for generating the 4d solutions with four independent charges. The gauged pairwise-equal charges solution comes up by subtraction of the scalar potential from the ungauged bosonic Lagrangian with $\varphi_2 = \varphi_3 = \chi_2 = \chi_3 = 0$.

where

$$\begin{aligned}
\Delta_r &= r^2 + a^2 - 2mr + \frac{1}{\ell^2} r_1 r_2 (r_1 r_2 + a^2), \\
\Delta_\theta &= 1 - \frac{a^2}{\ell^2} \cos^2 \theta, \quad W = r_1 r_2 + a^2 \cos^2 \theta, \\
r_i &= r + 2ms_i^2 = r + q_i, \quad \Xi = 1 - \frac{a^2}{\ell^2}, \\
s_i &= \sinh \delta_i, \quad c_i = \cosh \delta_i,
\end{aligned} \tag{2.4}$$

also axion, dilaton and gauge potentials are read

$$\begin{aligned}
e^\varphi &= \frac{r_1^2 + a^2 \cos^2 \theta}{W}, & \chi_1 &= \frac{a(r_2 - r_1) \cos^2 \theta}{r_1^2 + a^2 \cos^2 \theta}, \\
A_{(1)1} &= \frac{2\sqrt{2}m(dt - a \sin^2 \theta \Xi^{-1} d\phi)}{W} s_1 c_1 r_2, & A_{(1)2} &= \frac{2\sqrt{2}m(dt - a \sin^2 \theta \Xi^{-1} d\phi)}{W} s_2 c_2 r_1.
\end{aligned} \tag{2.5}$$

The coordinate change $\tilde{\phi} = \phi + ag^2 t$ yields an asymptotically static frame (ASF). The Hawking temperature, entropy, angular velocity and electrostatic potentials on the horizon (in the asymptotically rotating frame) are

$$\begin{aligned}
T_H &= \frac{r_+^2 - a^2 + a^2/\ell^2(r_+^2 - q_1 q_2) + (r_+ + q_1)(r_+ + q_2)(3r_+^2 + q_1 r_+ + q_2 r_+ - q_1 q_2)/\ell^2}{4\pi r_+ [(r_+ + q_1)(r_+ + q_2) + a^2]}, \\
S &= \frac{\pi[(r_+ + q_1)(r_+ + q_2) + a^2]}{\Xi}, & \Omega &= \frac{\Xi a}{\ell^2((r_+ + q_1)(r_+ + q_2) + a^2)}, \\
\Phi_1 &= \Phi_2 = \frac{2ms_1 c_1 (r_+ + q_2)}{(r_+ + q_1)(r_+ + q_2) + a^2}, & \Phi_3 &= \Phi_4 = \frac{2ms_2 c_2 (r_+ + q_1)}{(r_+ + q_1)(r_+ + q_2) + a^2},
\end{aligned} \tag{2.6}$$

where r_+ is the largest root of $\Delta_r = 0$ as the outer horizon.

The charge quantities including mass, angular momentum and pairwise electric potentials were constructed in [22] are

$$\begin{aligned}
E &= \frac{m}{\Xi^2} (1 + s_1^2 + s_2^2) = \frac{2m + q_1 + q_2}{2\Xi^2}, \\
J &= \frac{ma}{\Xi^2} (1 + s_1^2 + s_2^2) = \frac{a(2m + q_1 + q_2)}{2\Xi^2}, \\
Q_1 &= Q_2 = \frac{ms_1 c_1}{2\Xi}, & Q_3 &= Q_4 = \frac{ms_2 c_2}{2\Xi}.
\end{aligned} \tag{2.7}$$

In the next subsection we use the ultra-spinning (super-entropic) limit upon the metric (2.3) in order to obtain a new charged-AdS black hole solution in gauged supergravity.

This novel ultra-spinning (super-entropic) limit can be interpreted as a simple method to generate a new black hole solution in which the rotation parameter a , reaches to its maximum amount, equal to the AdS radius ℓ . This procedure limit consists of three steps.

- i) Transforming metric to an asymptotic rotating frame to avoid a singular metric in ultra-spinning limit. We then need only define a new azimuthal coordinate $\varphi = \phi/\Xi$ afterward.
- ii) This rotation has to be boosted effectively to the speed of light namely by taking the $a \rightarrow \ell$ limit.
- iii) In final step, we compactify the new azimuthal direction φ [1].

We note that employing an asymptotically rotating coordinate should be assumed as a crucial point in the ultra-spinning limit technique. The uniqueness of the choice of ARF to have a non-singular black hole solution has been discussed in [13]. In fact, avoiding to start from an ARF leads us to a singular limit.

2.1 Ultra-spinning limit

Since the metric (2.3) is already written in an asymptotically rotating frame, we therefore need only to introduce a new azimuthal coordinate $\varphi = \phi/\Xi$ followed by taking the limit $a \rightarrow l$. Hence we straightforwardly obtain the following solution

$$ds^2 = -\frac{\tilde{\Delta}_r}{\tilde{W}}(dt - \ell \sin^2 \theta d\varphi)^2 + \tilde{W}\left(\frac{dr^2}{\tilde{\Delta}_r} + \frac{d\theta^2}{\sin^2 \theta}\right) + \frac{\sin^4 \theta}{\tilde{W}}[\ell dt - (r_1 r_2 + \ell^2)d\varphi]^2, \quad (2.8)$$

where $\tilde{\Delta}_r$ and \tilde{W} are given by (2.4) as $a \rightarrow l$. To exclude a conical singularity in φ direction, one can identify it with period $2\pi/\Xi$. Since the new azimuthal coordinate φ is non-compact, we now compactify it by requiring

$$\varphi \sim \varphi + \mu, \quad (2.9)$$

where the parameter μ is dimensionless. Note, there is still an axial Killing vector ∂_φ in the new coordinate direction φ . Therefore, it is straightforward to show that the obtained metric (2.8) appears as a new exact solution. Also we can easily find dilaton, axion and gauge fields in this limit as

$$\begin{aligned} e^{\tilde{\varphi}} &= \frac{r_1^2 + \ell^2 \cos^2 \theta}{\tilde{W}}, & \tilde{\chi}_1 &= \frac{\ell(r_2 - r_1) \cos \theta}{r_1^2 + \ell^2 \cos^2 \theta}, \\ \tilde{A}_{(1)1} &= \frac{2\sqrt{2}ms_1c_1r_2(dt - l \sin^2 \theta d\varphi)}{\tilde{W}}, & \tilde{A}_{(1)2} &= \frac{2\sqrt{2}ms_2c_2r_1(dt - l \sin^2 \theta d\varphi)}{\tilde{W}}. \end{aligned} \quad (2.10)$$

Horizon geometry: To ensure that the new solution (2.8) is describing a black hole, we examine the largest root of $\tilde{\Delta}_r$ in (2.8) which is supposed to demonstrate the location of the horizon as r_+ . It is required now to check $\tilde{\Delta}'_r \geq 0$. We therefore find following mass bound

$$m \geq m_0 \equiv \frac{\sqrt{3}}{18\ell^2}[(q_1 - q_2)^2 - 4\ell^2]^{\frac{3}{2}} - \frac{q_1 + q_2}{2}, \quad |q_2 - q_1| \geq 2\ell. \quad (2.11)$$

For $m > m_0$ a horizon exists, while for $m < m_0$ there exists a naked singularity. Also to ensure that our obtained geometry would be free of any closed timelike curves (CTC) we examine $g_{\varphi\varphi} \geq 0$

$$g_{\varphi\varphi} = \frac{\ell^2 \sin^4 \theta [(2m + r_+)(q_1 + q_2) + q_1 q_2]}{(r_+ + q_1)(r_+ + q_2) + \ell^2 \cos^2 \theta}. \quad (2.12)$$

Apparently $g_{\varphi\varphi}$ is positive in whole spacetime.

Now, let us take a deeper look at the geometry of the horizon. The induced metric on a constant (t, r) surface yields,

$$ds_h^2 = \frac{\tilde{W}_+}{\sin^2 \theta} d\theta^2 + \sin^4 \theta \frac{((r_+ + q_1)(r_+ + q_2) + \ell^2)^2}{\tilde{W}_+} d\varphi^2. \quad (2.13)$$

where $\tilde{W}_+ = \tilde{W}|_{r_+}$. However, this geometry seems to be singular in $\theta = 0$ and π , we will show that the symmetry axis $\theta = 0, \pi$ is actually not part of the spacetime. For a precise study about these poles, one can probe the metric in the small $\theta = 0$ limit by introducing the change of variables similar to [13],

$$k = l(1 - \cos \theta), \quad (2.14)$$

Horizon metric (2.13) for small k becomes

$$ds_h^2 = (r_+ + q_1)(r_+ + q_2) \left[\frac{dk^2}{4k^2} + \frac{4k^2}{\ell^2} d\varphi^2 \right], \quad (2.15)$$

which shows clearly a metric of constant negative curvature on a quotient of the hyperbolic space \mathbb{H}^2 . Note, due to the symmetry, the $\theta = \pi$ limit gets the same result. Thus, there is no true singularity at these two points, but some sort of boundaries. Therefore topologically, the event horizon is sphere with two punctures, which implies that our obtained new black hole enjoys a finite area but non-compact horizon.

In order to visualize the geometry of the horizon (2.13) we can embed it in Euclidean 3-space as a surface of revolution [11]. We then identify the induced metric on the horizon (2.13) by using a flat metric in cylindrical coordinates as,

$$ds_3^2 = dz^2 + dR^2 + R^2 d\Phi^2, \quad (2.16)$$

and consider $z = z(\theta)$, $R = R(\theta)$. Setting $\Phi = \frac{2\pi}{\mu}\varphi$, hence one gets

$$R^2(\theta) = \left(\frac{\mu}{2\pi} \right) \frac{[(r_+ + q_1)(r_+ + q_2) + \ell^2]^2}{(r_+ + q_1)(r_+ + q_2) + \ell^2 \cos^2 \theta} \sin^4 \theta, \quad (2.17)$$

$$\left(\frac{dz(\theta)}{d\theta} \right)^2 + \left(\frac{dR(\theta)}{d\theta} \right)^2 = \frac{(r_+ + q_1)(r_+ + q_2) + \ell^2 \cos^2 \theta}{\sin^2 \theta}, \quad (2.18)$$

which is a differential equation for $dz/d\theta$. We integrated (2.18) numerically for the values $\ell = 1$, $\mu = 2\pi$, and $r_+ = 0.8$, by choosing different amounts of q_1 and q_2 . The resulting surfaces of revolution are shown in Fig. 1.

Conformal Boundary : Let us here, find the conformal boundary of our obtained new black hole solution (2.8) (the conformal factor is ℓ^2/r^2)

$$ds_{bdry}^2 = -dt^2 + \ell \sin^2 \theta dt d\varphi + \frac{1}{\sin^4 \theta} d\theta^2. \quad (2.19)$$

It is appear that the new coordinate φ would be a null one on the conformal boundary. Now we study this metric near the pole $\theta = 0$ using (2.14), For small k we have then

$$ds_{bdry}^2 = -dt^2 + 2k dt d\varphi + \frac{1}{4k^2} dk^2. \quad (2.20)$$

This metric can be interpreted as an AdS_3 written in Hopf-like fibration over \mathbb{H}^2 . It means again that the poles $\theta = 0, \pi$ are not parts of the spacetime indeed, and they are being removed from the boundary. However, answering to what happens precisely at $\theta = 0$ and π , we should study on the behaviour of the geodesics in whole spacetime. To do that any outgoing null geodesics from inside the horizon may not be reached to the symmetry axis $\theta = 0$ in a finite affine parameter, similar to

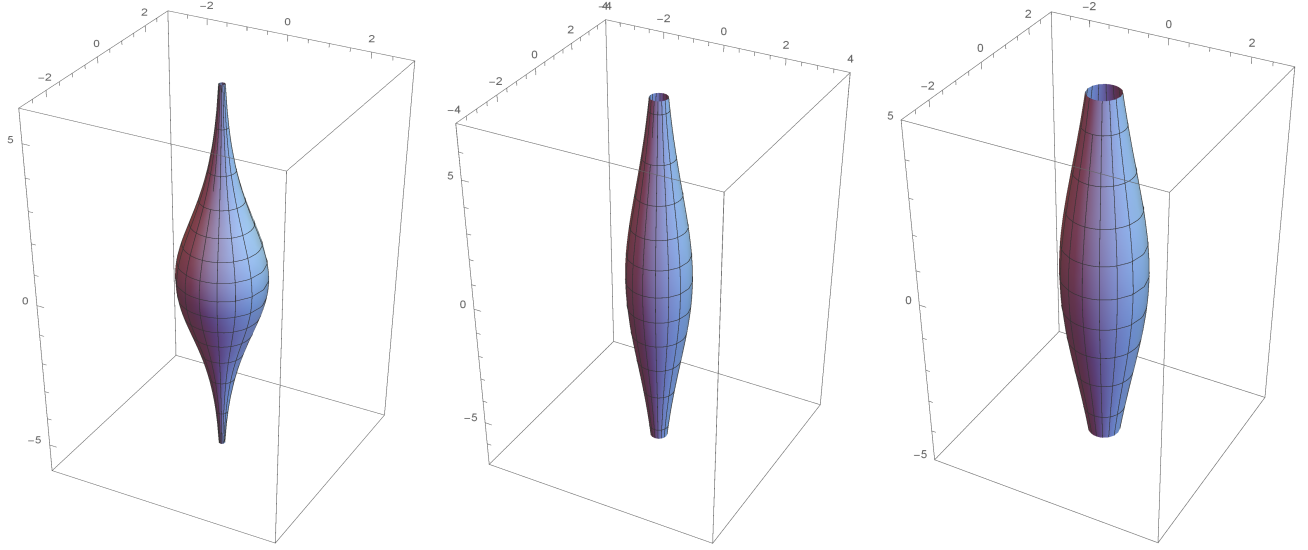


Fig. 1. Horizon embeddings in 4d. Plots show 2-dimensional non-compact horizons embedded in \mathbb{R}^3 as surface of revolution, displaying topology of a sphere with two punctures. We have set $\mu = 2\pi$, $r_+ = 0.8$ and $\ell = 1$ for all diagrams, with $q_1 = 10$, $q_2 = 0$ (left), $q_1 = 10$, $q_2 = 5$ (middle), and $q_1 = 3$, $q_2 = 6$ (right).

the strategy taken in [13]. This study of course, is not in scope of this paper for now, we postpone studying on it to another further work.

Recalling, AdS/CFT viewpoint hints to explore a dual 3-dimensional CFT which exists on the AdS_3 conformal boundary (2.20). We note, depending on the choice of an AdS coordinate slicing, different manifolds may be achieved where the corresponding QFT lives on it. Each coordinate covers the whole or a part of AdS boundary indeed, for instance, in global coordinate the dual CFT lives on $R \times S^p$ geometry, while in Poincaré patch the CFT exists on $R^{p,1}$ manifold. Therefore, in the slicing (2.20) one may expect the dual CFT resides on an AdS_3 geometry. These CFTs can be viewed as a possible deformations around its conformal fixed point which may be related to each other by Wilsonian renormalization group flow equation.

Thermodynamic quantities : Although, the event horizon of the new obtained ultra-spinning black hole (2.8) is non-compact, it has a finite area and entropy as

$$S = \frac{\mu}{2} [(r_+ + q_1)(r_+ + q_2) + \ell^2]. \quad (2.21)$$

For solution (2.8) we extract the Hawking temperature, electrostatic potentials and angular velocity on the horizon. Our results are

$$\begin{aligned} T_H &= \frac{2r_+^2 - \ell^2 - q_1 q_2 + (r_+ + q_1)(r_+ + q_2)(3r_+^2 + q_1 r_+ + q_2 r_+ - q_1 q_2)/\ell^2}{4\pi r_+ [(r_+ + q_1)(r_+ + q_2) + \ell^2]}, \\ \Phi_1 &= \Phi_2 = \frac{\sqrt{q_1(2m + q_1)}(r_+ + q_2)}{(r_+ + q_1)(r_+ + q_2) + \ell^2}, & \Phi_3 &= \Phi_4 = \frac{\sqrt{q_2(2m + q_2)}(r_+ + q_1)}{(r_+ + q_1)(r_+ + q_2) + \ell^2}, \\ \Omega &= \frac{\ell}{(r_+ + q_1)(r_+ + q_2) + \ell^2}. \end{aligned} \quad (2.22)$$

The electric potential is computed using $\Phi^I = K^\mu A_\mu^I$, where $K^\mu = \partial_t + \Omega_s \partial_\varphi$ is the null Killing vector

generating the horizon. We calculated then the following conserved charges as

$$\begin{aligned} E &= \frac{2m + q_1 + q_2}{2}, & J &= \frac{\ell(2m + q_1 + q_2)}{2} \\ Q_1 &= Q_2 = \frac{\sqrt{q_1(2m + q_1)}}{4}, & Q_3 &= Q_4 = \frac{\sqrt{q_2(2m + q_2)}}{4}. \end{aligned} \quad (2.23)$$

Angular momentum can be obtained by using the Komar integral based on the angular Killing vector ∂_φ , also the electric charges are calculated by using the Gaussian integrals. For computing mass we use the conformal approach of Ashtekar, Magnon and Das (AMD) [23]. In the AMD method the electric part of the Weyl tensor plays an essential role, whose spatial conformal boundary integral gives us the mass. It was shown that for a large variety of gauged supergravity black holes this mass exactly agrees with the first law of thermodynamics calculation [24].

Extremality under ultra-spinning limit: For an extremal black hole the inner and outer horizons coincide to a single horizon at r_0 , with vanishing Hawking temperature and generically non-zero entropy.

Here, in order to answer whether extremality condition will be preserved under ultra-spinning limit or not, we consider two different approaches,

- i) Finding the extremality condition of a given black hole and then applying the ultra-spinning limit.
- ii) Performing the ultra-spinning limit to a given geometry firstly, and then finding the extremality condition of the obtained ultra-spinning black hole afterwards.

Firstly, let us to start from the situation i), the extremality conditions of metric (2.3) can be found by using (2.6) and (2.4), imposing $T_H|_{r=r_0} = 0$ and $\Delta_r = 0$. Hence degenerate horizon r_0 reads

$$r_0 = \frac{1}{6} \left[(3\mathcal{B} + 3\sqrt{\mathcal{A}^3 + \mathcal{B}})^{1/3} - \frac{\mathcal{A}}{(3\mathcal{B} + 3\sqrt{\mathcal{A}^3 + \mathcal{B}})^{1/3}} - 3(q_1 + q_2) \right], \quad (2.24)$$

where $\mathcal{A} = 3^{2/3} [2(a^2 + \ell^2) - (q_1 - q_2)^2]$, and $\mathcal{B} = 9\ell^2(2m + q_1 + q_2)$. Also the following constraints between parameters of solution and r_0 are achieved

$$\begin{aligned} q_1 &= \frac{\sqrt{(a^2 q_2 - 4r_0^2(r_0 + q_2))^2 - 4(r_0^2 - q_2^2)[-a^2 \ell^2 + r_0^2(a^2 + \ell^2 + q_2^2 + 4q_2 r_0 + 3r_0^2)]}}{2(r_0^2 - q_2^2)} \\ &+ \frac{a^2 q_2 - 4r_0^2(r_0 + q_2)}{2(r_0^2 - q_2^2)^2}, \\ m &= \frac{a^4 q_2 + 4\ell^2 q_2(r_0 - q_2) + 2(r_0 + q_2)^2 [2r_0(r_0 + q_2)^2 + \chi] - a^2 [4\ell^2(q_2 - r_0) + 4r_0(r_0 + q_2)^2 + \chi]}{4\ell^2(r_0 - q_2)^2}, \end{aligned} \quad (2.25)$$

where $\chi = \sqrt{a^4 q_2^2 + 4(r_0 + q_2)(r_0^2(q_2 + r_0)((q_2 + r_0)^2 - a^2) + 4\ell^2(q_2 - r_0)(r_0^2 - a^2))}$.

Now, upon taking the limit $a \rightarrow l$, one easily gets

$$\begin{aligned} q_1 &= \frac{\ell^2 q_2 - 4q_2 r^2 - 4r^3 + \sqrt{4r_0^2(q_2 + r_0)^4 + \ell^4(-3q_2^2 + 4r_0^2) - 8\ell^2 r_0^3(q_2 + r_0)}}{2(r_0^2 - q_2^2)}, \\ m &= \frac{\ell^4(-3q_2 + 4r_0) + 2(q_2 + r_0)^2 [2r_0(q_2 + r_0)^2 + \chi_s] - \ell^2 [4r_0^2(3q_2 + r_0) + \chi_s]}{4\ell^2(r_0 - q_2)^2}, \end{aligned} \quad (2.26)$$

where $\chi_s = \sqrt{4r_0^2(r_0 + q_2)^4 + \ell^4(4r_0^2 - 3q_2^2) - 8\ell^2 r_0^3(r_0 + q_2)}$. The horizon also obtained as

$$r_0 = \frac{1}{6} \left[(3\mathcal{B} + 3\sqrt{\tilde{\mathcal{A}}^3 + \mathcal{B}})^{1/3} - \frac{\tilde{\mathcal{A}}}{(3\mathcal{B} + 3\sqrt{\tilde{\mathcal{A}}^3 + \mathcal{B}})^{1/3}} - 3(q_1 + q_2) \right], \quad (2.27)$$

where $\tilde{\mathcal{A}} = 3^{2/3} [2\ell^2 - (q_1 - q_2)^2]$.

Now, we examine the approach ii). In this way we only need to find the extremality condition of our obtained ultra-spinning black hole (2.8) by using (2.22). Our resulting conditions are exactly same as (2.26) and (2.27). It means that the extremality conditions commute with the ultra-spinning limit for 4-dimensional $U(1)^4$ gauged supergravity (2.3) black hole.

2.2 Near horizon geometry

Extremal black holes have several interesting characteristics. One of them is that the near horizon region of extremal black holes intuitively can be interpreted as an isolated geometry and isolated thermodynamical system. It has been discussed in [25, 26] that focusing on the near horizon geometry of extremal black holes arises a new class of solutions to the same theory of gravity, which their conserved charges are the same as the ones's of the original black hole. Near horizon extremal geometries (NHEG) have no horizon and no singularity unlike black holes, with enhanced $SL(2, \mathbb{R}) \times U(1)^N$ symmetry, and their asymptotic behaviours are different. Analogues to black hole thermodynamics, the laws of NHEG dynamics have been derived in [27], in which despite the absence of event horizon in NHEG there is an entropy associated to them as a Noether charge. However in this case system cannot be excited while it keeps $SL(2, R)$ isometry [28, 29].

In this section we try to find the near horizon geometry of the obtained ultra-spinning black hole (2.8) in the extremal limit. Now we can write the near horizon expansion as

$$\tilde{\Delta}_r = X(r - r_0)^2 + \mathcal{O}(r - r_0)^3,$$

where

$$X = 2 + \frac{1}{\ell^2} (6r_0^2 + 6q_1 r_0 + 6q_2 r_0 + q_1^2 + q_2^2 + 4q_1 q_2). \quad (2.28)$$

Now, in order to find the near horizon geometry in the extremal limit, we use following dimensionless coordinate changes $(\hat{t}, \rho, \hat{\theta}, \hat{\varphi})$ to extract the NHEG as an exact solution.

$$r = r_0(1 + \lambda \hat{r}), \quad \varphi = \hat{\varphi} + \Omega_H^0 \hat{t}, \quad t = \frac{\hat{t}}{2\pi T'_H r_0 \lambda}, \quad \hat{\theta} = \theta. \quad (2.29)$$

Here, the quantity Ω_H^0 is the angular velocity on the horizon in extremal limit, which is a shift for the coordinate φ to attain the co-moving coordinate of the horizon. In these new coordinates the killing vector $\xi = \lambda/r_0 \partial_t$ is the horizon generator. Also the quantity $T_H'^0$ is defined as

$$T_H'^0 = \left. \frac{\partial T_H}{\partial \hat{r}_+} \right|_{\hat{r}_+ = r_0}. \quad (2.30)$$

Now upon taking the limit $\lambda \rightarrow 0$, the near-horizon metric reads,

$$\begin{aligned} ds^2 &= \frac{\tilde{W}_0}{X} \left(-\hat{r}^2 d\hat{t}^2 + \frac{d\hat{r}^2}{\hat{r}^2} \right) + \frac{\tilde{W}_0}{\sin^2 \theta} d\hat{\theta}^2, \\ &+ \frac{\sin^4 \theta}{\ell^2 \tilde{W}_0} \left[(r_0 + q_1)(r_0 + q_2) + \ell^2 \right]^2 (d\hat{\varphi} + k\rho d\hat{t})^2, \end{aligned} \quad (2.31)$$

where

$$k = \frac{\ell(2r_0 + q_1 + q_2)}{X[(r_0 + q_1)(r_0 + q_2) + \ell^2]}, \quad (2.32)$$

and $\tilde{W}_0 = \tilde{W}|_{\hat{r}=r_0}$. This metric can be viewed as the direct product of $\text{AdS}_2 \times S^2$, in which the AdS sector is written here by Poincaré-type coordinates (\hat{t}, \hat{r}) . It appears that the NHEG of the ultra-spinning black hole (2.8) gives the well-known result which contains an AdS_2 sector. The metric (2.31) similar to the horizon geometry, seems to be singular in $\theta = 0, \pi$. We showed in previous subsections that these points are not truly singularities but they are some kinds of boundaries. Therefore the S^2 sector of NHEG (2.31) inherits the non-compactness characteristic and then it topologically would be a sphere with two punctures.

Metric (2.31) can be cast into the general form of the NHEG constructed in [8] which is calculated for the most general gauged extremal and stationary supergravity black holes as what follows

$$\begin{aligned} ds^2 &= \Gamma(\theta) \left[-\rho^2 dt^2 + \frac{d\rho^2}{\rho^2} + \alpha(\theta) d\theta^2 \right] + \gamma(\theta) (d\phi + k\rho dt)^2, \\ A^I &= f(\theta) (d\phi + k\rho dt) + e^I r dt. \end{aligned} \quad (2.33)$$

Where $\Gamma(\theta), \gamma(\theta)$ and $\alpha(\theta)$ are those extracted from (2.31). Additionally, to explore the behaviour of the gauge fields in the near horizon limit (2.29) we should carry out a new gauge transformation $A^I \rightarrow A^I + d\Lambda^I$ on parameter Λ [30].

$$\Lambda = \frac{\Phi_e^{I \text{ext}}}{\lambda} r_0 \hat{t}, \quad (2.34)$$

where $\Phi_e^{I \text{ext}}$ are the electrostatic potentials on the horizon in the extremal limit. This gauge transformation can be realized as a simple embeddings of $U(1)$ gauge field in a higher-dimensional auxiliary spacetime [30].

2.3 A quick review on Kerr/CFT

Here, we test the conjecture Kerr/CFT correspondence and its extensions for our non-compactness horizon black hole. Kerr/CFT explains that there exists a duality between the near horizon states of a 4d extremal kerr black hole and a certain $d = 2$ chiral conformal field theory [6]. The general NHEG (2.33) by imposing a set of consistent boundary conditions admits an enhanced $SL(2, R)_R \times U(1)_L^n$ isometry group [6, 31]. Indeed it includes all the symmetries of AdS_2 plus translations in φ^i coordinates. The symmetry generators are given by the following Killing vectors fields

$$\xi_1 = \partial_t, \quad \xi_2 = t\partial_t - r\partial_r, \quad \xi_3 = \frac{1}{2} \left(\frac{1}{r^2} + t^2 \right) \partial_t - tr\partial_r - \sum_{i=1}^n \frac{k_i}{r} \partial_\phi^i, \quad \bar{\xi}_i = \partial_\phi^i, \quad (2.35)$$

One can write a relation between the $SL(2, R)$ and $U(1)^n$ symmetry generators [27]

$$n^i \xi_i = k^i \bar{\xi}_i, \quad (2.36)$$

where n_i is the unit vector normal to AdS_2 .

Therefore, because of the exact similarity between NHEG of our solutions (2.29) and (2.33), one can deduce that the metric (2.31) is invariant under diffeomorphisms generated by $\bar{\xi}_1$ and $\xi_{1,2,3}$. Therefore an asymptotic symmetry group (ASG) associated to every consistent set of boundary

conditions can be found. Asymptotic symmetries of (2.31) may contain diffeomorphisms ζ and a $U(1)$ gauge transformation such that [7]

$$\delta_\zeta g_{\mu\nu} = \mathcal{L}_\zeta g_{\mu\nu}, \quad \delta_\zeta A_\mu = \mathcal{L}_\zeta A_\mu, \quad \delta_\Lambda A = d\Lambda. \quad (2.37)$$

The infinitesimal field variations are defined by $a_\mu = \delta A_\mu$ and $h_{\mu\nu} = \delta g_{\mu\nu}$. The combined transformation (ζ, Λ) has an associated charge $Q_{\zeta, \Lambda}$ defined in [31], which generates the symmetry (ζ, Λ) under Dirac brackets.

The charge $Q_{\zeta, \Lambda}$ must be finite for all field variations, required to satisfy a consistent boundary condition. Now we choose same boundary conditions as in [6]. The method used in [6] is assuming ∂_ϕ proportional to zero mode of a non-trivial Virasoro algebra, as well as indicating the boundary conditions in terms of power law falloff of the components of the metric fluctuations $h_{\mu\nu}$ as,

$$\begin{pmatrix} h_{tt} = \mathcal{O}(r^2) & h_{t\varphi} = \mathcal{O}(1) & h_{t\theta} = \mathcal{O}(1/r) & h_{tr} = \mathcal{O}(1/r^2) \\ & h_{\varphi\varphi} = \mathcal{O}(1) & h_{\varphi\theta} = \mathcal{O}(1/r) & h_{\varphi r} = \mathcal{O}(1/r) \\ & & h_{\theta\theta} = \mathcal{O}(1/r) & h_{\theta r} = \mathcal{O}(1/r^2) \\ & & & h_{rr} = \mathcal{O}(1/r^3) \end{pmatrix}, \quad (2.38)$$

also boundary condition for the gauge field reads

$$a_\mu = \mathcal{O}(r, \frac{1}{r}, 1, 1/r^2). \quad (2.39)$$

The most general diffeomorphisms ζ_m which preserve above boundary conditions are given by

$$\zeta_\epsilon = \epsilon(\phi)\partial_\psi - r\epsilon'(\phi)\partial_r, \quad \bar{\zeta} = \partial_t, \quad (2.40)$$

and the Virasoro algebra can be extracted as

$$i[\zeta_m, \zeta_n] = (m - n)\zeta_{m+n}, \quad \epsilon_n(\phi) = -e^{-in\phi}. \quad (2.41)$$

The gauge field transformation A under ζ_ϵ does not satisfy the boundary condition (2.39). So to restore $\delta A_\varphi = \mathcal{O}(1/r)$ we must impose a suitable compensating $U(1)$ gauge transformation as $\Lambda = -f(\theta)\epsilon(\phi)$ [7]. The Virasoro algebra with vanishing central charge of ASG reads

$$\begin{aligned} [\Lambda_m, \Lambda_n]_\zeta &= \zeta_m^\mu \partial_\mu \Lambda_n - \zeta_n^\mu \partial_\mu \Lambda_m, \\ i[(\zeta_m, \Lambda_m), (\zeta_n, \Lambda_n)]_\zeta &= (m - n)(\zeta_{m+n}, \Lambda_{m+n}). \end{aligned} \quad (2.42)$$

The gauge transformation (ζ_n, Λ_n) and charges Q_n associated to them satisfy a similar algebra up to a central extension. Using (2.33) the Dirac brackets algebra of Q_n can be derived as [31, 7]

$$\begin{aligned} i\{Q_{\zeta_\epsilon, \Lambda}, Q_{\zeta_{\tilde{\epsilon}}, \tilde{\Lambda}}\}_{DB} &= iQ_{(\zeta_\epsilon, \Lambda), (\zeta_{\tilde{\epsilon}}, \tilde{\Lambda})} - \frac{ik}{16\pi} \int d\theta d\varphi \sqrt{\frac{\alpha(\theta)\gamma(\theta)}{\Gamma(\theta)}} \left(f(\theta)\Lambda\tilde{\epsilon}' + \Gamma(\theta)\epsilon'\tilde{\epsilon}'' \right. \\ &\quad \left. + [f(\theta)^2 + \gamma(\theta)]\epsilon\tilde{\epsilon}' - (\epsilon, \Lambda \leftrightarrow \tilde{\epsilon}, \tilde{\Lambda}) \right). \end{aligned} \quad (2.43)$$

The algebra of the charge Q_n associated to ASG generators (ζ_n, Λ_n) from (2.43) is

$$i\{Q_m, Q_n\}_{DB} = (m - n)Q_{m+n} + \frac{c}{12}(m^3 - \alpha m)\delta_{m+n, 0}. \quad (2.44)$$

We note that, α is a constant and can be absorbed to Q_0 . The central charge c_L has combinations of k^{grav} and k^{gauge} as $c = c_{grav} + c_{gauge}$. They can be nicely calculated in the manner described in [7, 8] as

$$c_{grav} = \frac{3k_i}{2\pi} \int_0^\pi d\theta \sqrt{\Gamma(\theta)\alpha(\theta)\gamma(\theta)}, \quad c_{gauge} = 0, \quad (2.45)$$

where the constants k_i 's are given by corresponding Frolov-Thorne temperatures as below

$$k_i = \frac{1}{2\pi T_i}. \quad (2.46)$$

Here, T_i 's referred to the left- and right-moving temperatures of the quantum field theory coming from the Frolov-Thorne vacuum, are restricted to extreme Kerr black hole [32, 6]. In order to determine these temperatures, one can use a scalar field expansion in terms of eigenstates of the asymptotic energy E and angular momentum J

$$\Phi = \sum_{E,J,l} \phi_{EJl} e^{-iE\hat{t}+iJ\hat{\varphi}} f_l(r, \theta). \quad (2.47)$$

In the near horizon region (2.29) we have

$$e^{-iE\hat{t}+iJ\hat{\varphi}} = e^{-i(E-\Omega_H^0 J)\hat{t}r_0/\lambda+iJ\hat{\varphi}} = e^{-in_R\hat{t}+n_L\hat{\varphi}}, \quad (2.48)$$

where $n_R = (E - \Omega_H^0 J)r_0/\lambda$ and $n_L = J$ are the left and right charges associated to ∂_ϕ and ∂_t in the near horizon metric. Having found diagonal density matrix associated to the vacuum by tracing (2.47) over the region inside the horizon. Boltzmann weighting factor in the energy-angular momentum eigenbasis reads as

$$e^{-\frac{(E-\Omega_H J)}{T_H}} \quad (2.49)$$

In the non-rotating ($\Omega_H = 0$) case, this vacuum reduces to the Hartle-Hawking vacuum. One can hence write the Boltzmann factor in terms of n_L , n_R and T_i as

$$e^{-\frac{(E-\Omega_H J)}{T_H}} = e^{-\frac{n_L}{T_L} - \frac{n_R}{T_R}}. \quad (2.50)$$

Dimensionless Frolov-Thorne temperatures T_i where defined firstly for higher-dimensional Kerr-AdS black holes in [33] are

$$T_L = \lim_{r_+ \rightarrow r_0} \frac{T_H^0}{\Omega_H^0 - \Omega_H} = -\frac{\partial T_H / \partial r_+}{\partial \Omega_H / \partial r_+} \Big|_{r_+=r_0}, \quad T_R = \frac{r_0}{\lambda} T_H \Big|_{r_+=r_0}. \quad (2.51)$$

For an extremal solution, The right-temperature vanishes, while there are $[(d-1)/2]$ left-temperatures associated to the 2d CFTs for each azimuthal φ coordinate. However, the extreme Kerr black hole has vanishing Hawking temperature, but quantum fields states outside the horizon live in a thermal state.

We remind that the (2.47) can be also extended using the first thermodynamics law for a charged rotating black hole

$$TdS = dM - (\Omega_H dJ + \Phi dQ). \quad (2.52)$$

Now by imposing the extremality constraint $T^{ex} dS = 0$, it gets finally

$$dS = \frac{dJ}{T_L} + \frac{dQ}{T_e} \quad (2.53)$$

also the Boltzmann factor can be extended to

$$e^{-n_R/T_R - n_L/T_L - Q/T_e}. \quad (2.54)$$

2.4 Ultra-spinning/CFT description

In this section we shall study Kerr/CFT correspondence for our obtained extremal ultra-spinning $U(1)^4$ gauged supergravity black hole (2.8). Our strategy beside confirming the existence of the CFT dual, will be to verify that the ultra-spinning limit and the Kerr-CFT limit commute with each other. It has been shown in [13] that the ultra-spinning limit commutes with the near horizon limit for 5d Kerr-AdS and minimal gauged supergravity black holes.

Here, we consider two different procedures to explore Kerr/CFT for our ultra-spinning black hole. i) Beginning with an extremal ultra-spinning black hole and then taking near horizon limit into account. ii) Finding the near horizon limit of an extremal black hole and then applying the ultra-spinning limit afterwards. These two different ways are clearly shown in Fig 2.

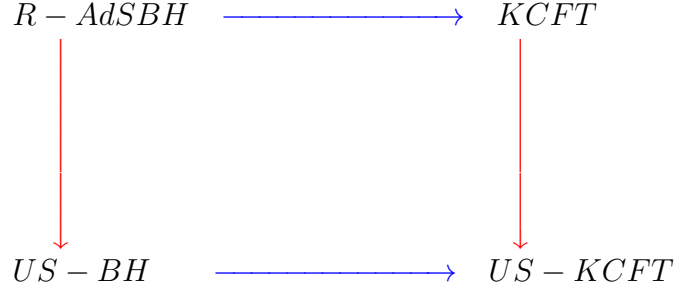


Fig. 2. This diagram illustrates two different order limits for a general rotating AdS black hole (R-AdS BH). Horizontal arrows (blue) represent the near horizon (NH) limit, to provide Kerr-CFT limit. Also the vertical ones (red) show the ultra-spinning (US) limit. We show that in both paths the resulting limit (US-KCFT) are exactly the same.

Let us here, start the lower path in Fig. 2. The resulting geometry is already obtained in (2.31). To find the central charge associated to metric (2.31), we need to write the extended version of the first law as

$$TdS = dM - \Omega_H dJ - \sum_i \Phi_i dQ_i - K d\mu. \quad (2.55)$$

Here, we consider μ as a thermodynamic parameter shown as a chemical potential [1]. It was explained in [34] that the compactified null length can be signified as a chemical potential. Since the new coordinate φ is a compact null coordinate on the conformal boundary which it becomes compactify by μ , then μ assumed to be a chemical potential, and its thermodynamic conjugate is denoted by K . Note, μ is a dimensionless quantity, so in Smarr formula there is no $K\mu$. Considering the extremality constraint on (2.55) gives

$$TdS = -[(\Omega_H - \Omega_H^{ex})dJ + \sum_{i=1}^4 (\Phi_i - \Phi_i^{ex})dQ_i + (K - K^{ex})d\mu]. \quad (2.56)$$

Then the Boltzmann factor (2.50) in extremal limit takes the following form

$$e^{-n_R/T_R - n_L/T_L - \sum_{i=1}^4 Q_i/T_{i,e} - \mu/T_\mu}, \quad (2.57)$$

where $n_R = (E - \Omega_H^{ex}J - \sum_i \Phi_i^{ex}Q_i - K^{ex}\mu)r_0/\lambda$ and $n_L = J$. Also T_R and T_L are given by (2.51). The Frolov-Thorne temperatures T_e and T_μ are defined as

$$T_{e,i} = -\frac{\partial T_H / \partial r_+}{\partial \Phi_i / \partial r_+} \Big|_{r_+=r_0}, \quad T_\mu = -\frac{\partial T_H / \partial r_+}{\partial K / \partial r_+} \Big|_{r_+=r_0}. \quad (2.58)$$

Now, using (2.51) and (2.58) we obtain following left- and right-moving temperatures

$$T_R = 0, \quad T_L = \frac{1}{2\pi k} = \frac{X[(r_0 + q_1)(r_0 + q_2) + \ell^2]}{2\pi\ell(2r_0 + q_1 + q_2)}, \quad (2.59)$$

and finally using (2.45) we find central charge as

$$c = 3\frac{\mu}{\pi} \frac{\ell(2r_0 + q_1 + q_2)}{X}. \quad (2.60)$$

We emphasize that the left-moving part of CFT identify the quantum field states on the near horizon region (2.31). Namely, in the extremal limit the vacuum state in the bulk, reduces to a mixed density matrix on the CFT side. The Boltzmann weighting factor reads

$$\rho = e^{-\frac{J}{T_L} - \sum_i \frac{Q}{T_{e,i}} - \frac{\mu}{T_\mu}}. \quad (2.61)$$

Indeed, CFT dual of the generalised Hartel-Hawking vacuum has temperature T_L . Now we apply the thermodynamic Cardy formula which counts microstates of an unitary and modular invariance CFT at large T and gives the entropy of CFT relating to its temperature and central charge [9].

$$S = \frac{\pi^2}{3} c_L T_L. \quad (2.62)$$

This relation confirms that c can be viewed as a measure of degrees of freedom and it determines the asymptotic density of states.

Now, using (2.59) and (2.60) we obtain the microscopic entropy of the dual CFT

$$S_{CFT} = \frac{\mu}{2} [(r_+ + q_1)(r_+ + q_2) + \ell^2], \quad (2.63)$$

which agrees precisely with the macroscopic Bekenstein-Hawking entropy (2.21). So this remarkable result confirms the Kerr/CFT conjecture again.

Now, we turn to the upper path in Fig. 2. At the first step we have to find the near horizon geometry of the metric (2.3) in the extremal limit, and then take the ultra-spinning limit on the NHEG. In [8] the NHEG of the metric (2.3) has been constructed. Its ultra-spinning limit can easily be calculated by replacing new coordinate $\varphi = \frac{\phi}{\Xi}$ and then setting $a \rightarrow l$ and finally compactifying the new coordinate φ with period μ . One can easily check that final result is exactly same as the (2.31). So we can conclude that their CFT dual and central charge are same too.

Hence these two results coming from procedures i) and ii) confirm that the ultra-spinning and the near horizon limits commute with each other for 4d gauged supergravity black hole solution.

Another issue which can be also discussed is, in black hole (2.8), the angular momentum J is not independent of the mass M . As we see in eq. (2.22) they are related by the chirality-type condition $M = J/l$. We follow thermodynamic interpretation like that in [12], thus we can define L_0 and \tilde{L}_0 in terms of M and J as

$$L_0 = \frac{(M + J/l)}{2}, \quad \tilde{L}_0 = \frac{(M - J/l)}{2}. \quad (2.64)$$

So in order to derive the Smarr formula in terms of L_0 and \tilde{L}_0 , we should consider $M = M(L_0, \tilde{L}_0, Q, \mu)$. Since in our case \tilde{L}_0 vanishes, the first law becomes

$$TdS = (1 - \Omega)dL_+ - \sum_i \Phi dQ - K d\mu, \quad (2.65)$$

the chirality condition $M = J/l$ states that the black hole microstates can be explained by chiral excitations of a 3d conformal field theory.

3 5-dimensional $U(1)^3$ gauged supergravity black holes

In this section we consider a charged rotating black hole which is constructed as a solutions of $SO(6)$ gauged five-dimensional supergarvity, whose relevant part of bosonic action is given by [35]

$$S_{5d} = \frac{1}{16\pi} \int d^5x \sqrt{-g} \left(R - \frac{1}{2} \partial \vec{\phi}^2 - \frac{1}{4} \sum_{i=1}^3 X_i^{-2} (F^i)^2 + \frac{4}{\ell^2} \sum_{i=1}^3 X_i^{-1} + \frac{1}{24} \epsilon_{ijk} \epsilon^{\mu\nu\rho\sigma\lambda} F_{\mu\nu}^i F_{\rho\sigma}^j A_{\lambda}^k \right), \quad (3.1)$$

where $\vec{\phi} = (\phi_1, \phi_2)$, and

$$X_1 = e^{-\frac{1}{\sqrt{6}}\phi_1 - \frac{1}{\sqrt{2}}\phi_2}, \quad X_2 = e^{-\frac{1}{\sqrt{6}}\phi_1 + \frac{1}{\sqrt{2}}\phi_2}, \quad X_3 = e^{\frac{2}{\sqrt{6}}\phi_1}. \quad (3.2)$$

A six parameter family of solutions including three electric charges, two angular momenta and mass, which is the most general asymptotically AdS_5 black hole solution to this theory has been constructed in [36]. Here, we consider the $U(1)^3$ Cartan subgroup of $SO(6)$ with three charge parameters δ_I that satisfy $\delta_1 = \delta_2 := \delta$ and $\delta_3 = 0$, as well as two independent rotation parameters. These four parameters family of solutions have been derived in [37]. Their metric is given by

$$\begin{aligned} ds^2 = & H^{-\frac{4}{3}} \left[-\frac{\Delta}{\rho^2} \left(dt - a \sin^2 \theta \frac{d\phi}{\Xi_a} - b \cos^2 \theta \frac{d\psi}{\Xi_b} \right)^2 \right. \\ & + \frac{C}{\rho^2} \left(\frac{ab}{f_3} dt - \frac{b}{f_2} \sin^2 \theta \frac{d\phi}{\Xi_a} - \frac{a}{f_1} \cos^2 \theta \frac{d\psi}{\Xi_b} \right)^2 + \frac{Z \sin^2 \theta}{\rho^2} \left(\frac{a}{f_3} dt - \frac{1}{f_2} \frac{d\phi}{\Xi_a} \right)^2 \\ & \left. + \frac{W \cos^2 \theta}{\rho^2} \left(\frac{b}{f_3} dt - \frac{1}{f_1} \frac{d\psi}{\Xi_b} \right)^2 \right] + H^{\frac{2}{3}} \left(\frac{\rho^2}{\Delta} dr^2 + \frac{\rho^2}{\Delta_\theta} d\theta^2 \right), \end{aligned} \quad (3.3)$$

where

$$\begin{aligned} H &= 1 + q/\rho^2, & \rho^2 &= r^2 + a^2 \cos^2 \theta + b^2 \sin^2 \theta, \\ f_1 &= a^2 + r^2, & f_2 &= b^2 + r^2, & f_3 &= f_1 f_2 + q r^2, \\ \Delta &= \frac{1}{r^2} (a^2 + r^2)(b^2 + r^2) - 2m + (a^2 + r^2 + q)(b^2 + r^2 + q)/\ell^2, \\ \Delta_\theta &= 1 - \frac{a^2}{\ell^2} \cos^2 \theta - \frac{b^2}{\ell^2} \sin^2 \theta, & C &= f_1 f_2 (\Delta + 2m - q^2/\rho^2), \\ Z &= -b^2 C + \frac{f_2 f_3}{r^2} [f_3 - \frac{r^2}{\ell^2} (a^2 - b^2)(a^2 + r^2 + q) \cos^2 \theta], \\ W &= -a^2 C + \frac{f_1 f_3}{r^2} [f_3 + \frac{r^2}{\ell^2} (a^2 - b^2)(b^2 + r^2 + q) \sin^2 \theta], \\ \Xi_a &= 1 - \frac{a^2}{\ell^2}, & \Xi_b &= 1 - \frac{b^2}{\ell^2}. \end{aligned} \quad (3.4)$$

The gauge and scalar fields are

$$\begin{aligned}
A^1 &= A^2 = \frac{\sqrt{q^2 + 2mq}}{\rho^2} (dt - a \sin^2 \theta \frac{d\phi}{\Xi_a} - \cos^2 \theta \frac{d\psi}{\Xi_b}), \\
A^3 &= \frac{q}{\rho^2} (b \sin^2 \theta \frac{d\phi}{\Xi_a} + a \cos^2 \theta \frac{d\psi}{\Xi_b}). \\
X_1 &= X_2 = H^{-\frac{1}{3}}, \quad X_3 = H^{\frac{2}{3}}
\end{aligned} \tag{3.5}$$

The metric (3.3) is written in asymptotic rotating frame. One can use a sort of coordinates to be asymptotically static (ASF) by taking $\phi = \tilde{\phi} + \frac{a}{\ell^2} t$ and $\psi = \tilde{\psi} + \frac{b}{\ell^2} t$.

The Hawking temperature, entropy and the electrostatic potentials on the horizon in the asymptotically static frame are

$$\begin{aligned}
T_H &= \frac{2r_+^6 + r_+^4(\ell^2 + a^2 + b^2 + 2q) - a^2 b^2 \ell^2}{2\pi r_+ \ell^2 [(r_+^2 + a^2)(r_+^2 + b^2) + q r_+^2]}, & S_{BH} &= \frac{\pi^2 [(r_+^2 + a^2)(r_+^2 + b^2) + q r_+^2]}{2r_+ \Xi_a \Xi_b}, \\
\Phi_1 &= \Phi_2 = \frac{\sqrt{q^2 + 2mq} r_+^2}{(a^2 + r_+^2)(b^2 + r_+^2) + q r_+^2}, & \Phi_3 &= \frac{a q b}{(a^2 + r_+^2)(b^2 + r_+^2) + q r_+^2}.
\end{aligned} \tag{3.6}$$

The Killing vector field that generates the Killing horizon is $K = \partial_t + \Omega_a^S \partial_\phi + \Omega_b^S \partial_\psi$, where Ω_a^S and Ω_b^S are the angular velocities on the horizon in (ASF). In ARF, the angular velocities on the horizon are written as

$$\Omega_a^R = \Omega_a^S - \frac{a}{\ell^2} = \frac{\Xi_a a (r_+^2 + b^2)}{(a^2 + r_+^2)(b^2 + r_+^2) + q r_+^2}, \quad \Omega_b^R = \Omega_b^S - \frac{b}{\ell^2} = \frac{\Xi_b b (r_+^2 + a^2)}{(a^2 + r_+^2)(b^2 + r_+^2) + q r_+^2}. \tag{3.7}$$

Conserved charges including two angular momenta, two electric charges and mass as mentioned in [37] are

$$\begin{aligned}
J_a &= \frac{\pi a (2m + q \Xi_b)}{4 \Xi_b \Xi_a^2}, & J_b &= \frac{\pi b (2m + q \Xi_a)}{4 \Xi_a \Xi_b^2}, \\
Q_1 &= Q_2 = \frac{\pi \sqrt{q^2 + 2mq}}{4 \Xi_a \Xi_b}, & Q_3 &= -\frac{\pi a b q}{4 \ell^2 \Xi_a \Xi_b}, \\
E &= \frac{\pi [2m(2\Xi_a + 2\Xi_b - \Xi_a \Xi_b) + q(2\Xi_a^2 + 2\Xi_b^2 + 2\Xi_a \Xi_b - \Xi_a^2 \Xi_b - \Xi_b^2 \Xi_a)]}{8 \Xi_a^2 \Xi_b^2}.
\end{aligned} \tag{3.8}$$

3.1 Ultra-spinning limit

We are now ready to perform the ultra-spinning limit on the metric (3.3), by following the same steps done in the previous section. This black hole rotates in two different directions ϕ and ψ , corresponding to the rotation parameters a and b . But we are only allowed to take the ultra-spinning limit for one azimuthal direction which is selected ϕ for us. Since the metric is already written in ARF, we begin by introducing a new azimuthal coordinate $\varphi = \phi/\Xi_a$. Then upon taking the $a \rightarrow \ell$ limit while we keep the parameter b fixed, we will have

$$\Delta_\theta = \Xi_b \sin^2 \theta, \tag{3.9}$$

hence we obtain the following new black hole solution as

$$\begin{aligned}
ds^2 = & \tilde{H}^{\frac{2}{3}} \left[-\frac{\tilde{\Delta}}{\tilde{\rho}^2} (dt - \ell \sin^2 \theta d\varphi - b \cos^2 \theta \frac{d\psi}{\Xi_b})^2 \right. \\
& + \frac{\tilde{C}}{\tilde{\rho}^2} (\frac{\ell b}{\tilde{f}_3} dt - \frac{b}{f_2} \sin^2 \theta d\varphi - \frac{\ell}{\tilde{f}_1} \cos^2 \theta \frac{d\psi}{\Xi_b})^2 + \frac{\tilde{Z} \sin^2 \theta}{\tilde{\rho}^2} (\frac{\ell}{\tilde{f}_3} dt - \frac{1}{f_2} d\varphi)^2 \\
& \left. + \frac{\tilde{W} \cos^2 \theta}{\tilde{\rho}^2} (\frac{b}{\tilde{f}_3} dt - \frac{\ell}{\tilde{f}_1} \frac{d\psi}{\Xi_b})^2 \right] + \tilde{H}^{\frac{2}{3}} (\frac{\tilde{\rho}^2}{\tilde{\Delta}} dr^2 + \frac{\tilde{\rho}^2}{\Xi_b \sin^2 \theta} d\theta^2), \tag{3.10}
\end{aligned}$$

where the \tilde{H} , \tilde{Z} , \tilde{W} , $\tilde{\Delta}$ and $\tilde{\rho}$ are given by (3.4) in which $a \rightarrow \ell$. Also the new coordinate φ can be compactified by $\varphi \sim \varphi + \mu$. The gauge potentials and scalar fields in this limit become

$$\begin{aligned}
A^1 = A^2 = & \frac{\sqrt{q^2 + 2mq}}{\tilde{\rho}^2} (dt - \ell \sin^2 \theta d\varphi - \cos^2 \theta \frac{d\psi}{\Xi_b}), \\
A^3 = & \frac{q}{\tilde{\rho}^2} (b \sin^2 \theta d\varphi + \ell \cos^2 \theta \frac{d\psi}{\Xi_b}), \\
X_1 = X_2 = & \tilde{H}^{-1/3}, \quad X_3 = \tilde{H}^{2/3}. \tag{3.11}
\end{aligned}$$

Now, it is easy to check that the metric (3.10) and fields (3.11) satisfy the equation of motion, and we can call (3.10) as a new exact gauged supergravity black hole solution.

It is worth to mention that, taking the ultra-spinning limit in the ψ direction (instead of ϕ) also is carried out by the same way. But it is impossible to take the ultra-spinning limit in both directions simultaneously, because the $g_{\theta\theta}$ component in metric (3.10) will diverge in $b \rightarrow \ell$ limit, and $1/\Xi_b$ divergence may not be absorbed into a new coordinate.

Horizon geometry: Here, we find the induced metric on the horizon

$$\begin{aligned}
ds_h^2 = & \tilde{H}^{-\frac{4}{3}} \left[\frac{\tilde{C}}{\tilde{\rho}^2} (\frac{b}{f_2} \sin^2 \theta d\varphi + \frac{\ell}{\tilde{f}_1} \cos^2 \theta \frac{d\psi}{\Xi_b})^2 + \frac{\tilde{Z} \sin^2 \theta}{\tilde{\rho}^2} \frac{1}{f_2^2} d\varphi^2 \right. \\
& \left. + \frac{\tilde{W} \cos^2 \theta}{\tilde{\rho}^2} (\frac{\ell}{\tilde{f}_1} \frac{d\psi}{\Xi_b})^2 \right] + \tilde{H}^{\frac{2}{3}} \frac{\tilde{\rho}^2}{\Xi_b \sin^2 \theta} d\theta^2. \tag{3.12}
\end{aligned}$$

As the 4-dimensional case, this metric seems to be ill-defined in $\theta = 0$, ($0 \leq \theta \leq \pi/2$). But to show that there is not any problem near this point, one can perform the change of coordinates $k = l(1 - \cos \theta)$, for small k , by which the horizon metric reads,

$$ds_h^2 = \frac{\rho_+}{\Xi_b} \left[\frac{dk^2}{4k^2} + 4k^2 \frac{(\ell^2 + r_+^2)^3 b^2 \Xi_b}{r_+^2 \ell^2 \rho_+^4} d\varphi^2 + \frac{4bkm}{\rho_+^4} d\varphi d\psi \right] + \frac{2m}{\rho_+^2 \Xi_b} d\psi^2. \tag{3.13}$$

For $\psi = \text{constant}$ slices, it reduces to a metric of constant negative curvature on a quotient of the hyperbolic space \mathbb{H}^2 , indicating that the horizon is non-compact. In Fig. 3 the embedding topology for constant ψ slices of the horizon, are displayed for $\mu = 2\pi$.

Conformal Boundary : To gain a deeper understanding of obtained geometry (3.10), let us here, take a look at its conformal boundary, with the conformal factor ℓ^2/r^2 .

$$\begin{aligned}
ds_{bdry}^2 = & -dt^2 + \ell \sin^2 \theta dt d\varphi + \frac{b \cos^2 \theta}{\Xi_b} dt d\psi + \frac{\ell^2 \cos^2 \theta}{\Xi_b} d\psi^2 \\
& + \frac{b\ell \sin^2 \theta \cos^2 \theta}{\Xi_b} d\varphi d\psi + \frac{\ell^2}{\Xi_b \sin^2 \theta} d\theta. \tag{3.14}
\end{aligned}$$

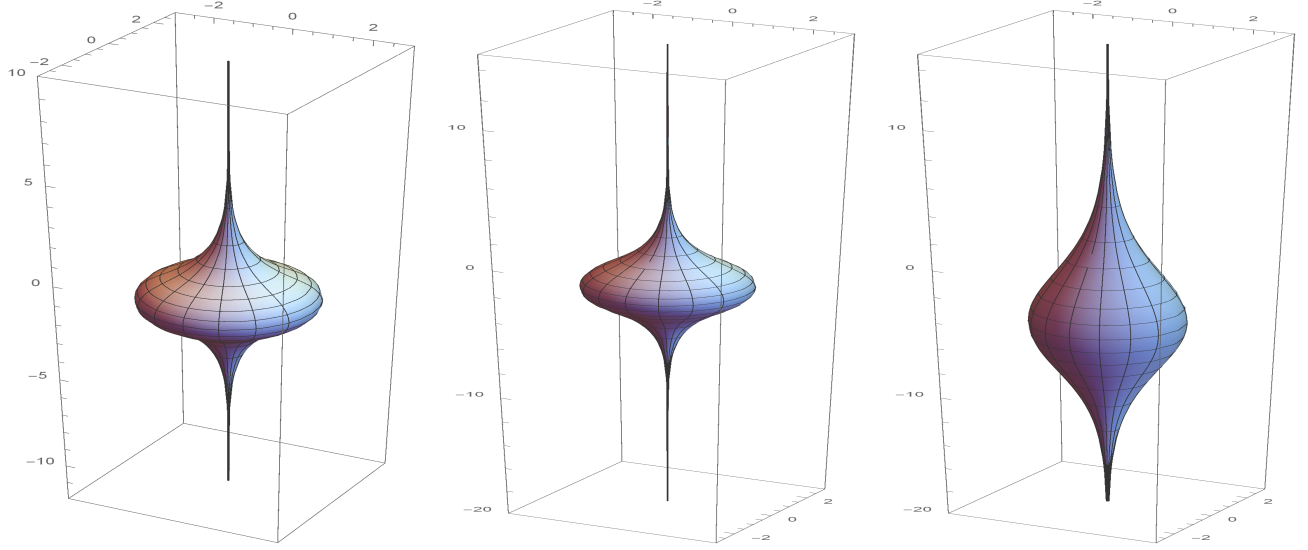


Fig. 3. Horizon embeddings in 5d. Plots show the 2-dimensional $\psi = \text{const.}$ slices horizon (3.12) embedded in \mathbb{R}^3 as a surface of revolution. we set $\mu = 2\pi$, $r_+ = 0.8$, $q = 10$ and $\ell = 1$ for all diagrams, and $b = 0.0$ (left), $b = 0.5$ (middle), as well as $b = 0.9$ (right). All cases show topology of a sphere with two punctures

The coordinate φ is a null coordinate on the conformal boundary. Moreover, as before we analyse this metric near the pole $\theta = 0$. Then for small k we have

$$ds_{bdry}^2 = -dt^2 + \frac{1}{\Xi_b} \left[\frac{dk^2}{4k^2} + 2kdt d\varphi + bdt d\psi + \ell^2 d\psi^2 + 2bk d\varphi d\psi \right] \quad (3.15)$$

which for $\psi = \text{constant}$ slices, it represents an AdS_3 written Hopf-like fibration over \mathbb{H}^2 . It means that the pole $\theta = 0$ is removed from the boundary and is not a part of the space-time indeed.

Thermodynamic quantities: For new obtained ultra-spinning black hole (3.10), we precisely derived the thermodynamic quantities including temperature, entropy and angular velocities on the horizon as well as electrostatic potentials on the horizon as

$$\begin{aligned} T_H &= \frac{2r_+^6 + r_+^4(2\ell^2 + b^2 + 2q) - b^2\ell^4}{2\pi r_+ \ell^2 [(r_+^2 + \ell^2)(r_+^2 + b^2) + qr_+^2]}, & S_{BH} &= \frac{\mu\pi[(r_+^2 + \ell^2)(r_+^2 + b^2) + qr_+^2]}{4r_+ \Xi_b}, \\ \Omega_a &= \frac{\ell(b^2 + r_+^2)}{b^2(\ell^2 + r_+^2) + r_+^2(\ell^2 + r_+^2 q)}, & \Omega_b &= \frac{b(r_+^4 + 2r_+^2 \ell^2 + r_+^2 q + \ell^4)}{\ell^2(r_+^2 + \ell^2)(r_+^2 + b^2) + q\ell^2 r_+^2}, \\ \Phi_1 &= \Phi_2 = \frac{r_+^2 \sqrt{q^2 + 2mq}}{(\ell^2 + r_+^2)(b^2 + r_+^2) + qr_+^2}, & \Phi_3 &= \frac{q\ell b}{(\ell^2 + r_+^2)(b^2 + r_+^2) + qr_+^2}. \end{aligned} \quad (3.16)$$

Extremality under ultra-spinning limit: To ensure that the extremality condition is preserved under the ultra-spinning limit for this 5d supergravity black hole, we similarly explore two different ways as mentioned in Fig. 2. So, we first obtain the extremality conditions of main metric (3.3) with a horizon at $r = r_0$ using Eq. (3.6) by imposing $\Delta = 0$ and $T_H|_{r=r_0} = 0$. We find two

following conditions

$$\begin{aligned} q &= -ab + \frac{r_0^2 \sqrt{a^2 + b^2 + \ell^2 + 2r_0^2}}{\ell}, \\ m &= \frac{1}{2} \frac{\ell^4 (a^2 + r_0^2)(b^2 + r_0^2) + r_0^2 (Y - a^2 \ell)(Y - b^2 \ell)}{r_0^2 \ell^4}, \end{aligned} \quad (3.17)$$

where $Y = abl - r_0^2 \sqrt{a^2 + b^2 \ell^2 + 2r_0^2}$.

Now, we find the extremality conditions of our obtained ultra-spinning black hole using (3.16)

$$\begin{aligned} q &= -b\ell + \frac{r_0^2 \sqrt{b^2 + 2(\ell^2 + r_0^2)}}{\ell} \\ m &= \frac{1}{2} \frac{\ell^4 (b^2 + r_0^2)(\ell^2 + r_0^2) + r_0^2 (Y_s - \ell^2(b - \ell))(Y_s + b\ell(b - \ell))}{r_0^2 \ell^4} \end{aligned} \quad (3.18)$$

where $Y_s = r_0^2(\ell + \sqrt{b^2 + 2(\ell^2 + r_0^2)})$. It is easy to check that Eq. (3.18) will be achieved by taking $a \rightarrow \ell$ limit on (3.17). Hence as the previous case, the extremality condition and the ultra-spinning limit commute with each other for five-dimensional $U(1)^3$ gauged supergravity class of black holes.

3.2 Near horizon geometry

For an extremal solution of ultra-spinning black hole (3.10) with a horizon at $r = r_0$, we have found the extremality conditions (3.18). Then one can write the near horizon expansion as

$$\Delta = \tilde{X}(r - r_0)^2 + \mathcal{O}(r - r_0)^3, \quad (3.19)$$

where

$$\tilde{X} = 2 + \frac{3\ell^2 b^2}{r_0^4} + \frac{1}{\ell^2} (6r_0^2 + b^2 + 2q). \quad (3.20)$$

To obtain the near horizon geometry, we use the coordinate changes similar to (2.29) as

$$\hat{r} = r_0(1 + \lambda r), \quad \hat{\varphi} = \varphi + \Omega_a^0 \hat{t}, \quad \hat{\psi}_2 = \psi + \Omega_b^0 \hat{t}, \quad \hat{t} = \frac{t}{2\pi T_H' r_0 \lambda}, \quad \hat{\theta} = \theta. \quad (3.21)$$

After applying the scaling parameter $\lambda \rightarrow 0$, the near horizon geometry in terms of the vielbeins takes the form of

$$ds^2 = H_0^{2/3} \frac{\rho_0^2}{\tilde{X}} \left(-\hat{r}^2 d\hat{t}^2 + \frac{d\hat{r}^2}{\hat{r}^2} \right) + F(\theta) d\hat{\theta}^2 + \sum_{i=1}^2 \hat{e}^i \hat{e}^i, \quad (3.22)$$

where $F(\theta) = H_0^{2/3} \frac{\rho_0^2}{\sin^2 \theta \Xi_b}$ and vielbeins are

$$\begin{aligned} \hat{e}^1 &= \alpha_1 \hat{e}_1 + \alpha_2 \hat{e}_2, & \hat{e}^2 &= \beta_1 \hat{e}_1 + \beta_2 \hat{e}_2, \\ \hat{e}_1 &= d\hat{\varphi} + k_\varphi \hat{r} d\hat{t}, & \hat{e}_2 &= d\hat{\psi} + k_\psi \hat{r} d\hat{t}, \end{aligned} \quad (3.23)$$

and

$$\begin{aligned}
k_\varphi &= \frac{2\ell[(r_0^2 + b^2)^2 + qb^2]}{\tilde{X}[(r_0^2 + \ell^2)(r_0^2 + b^2) + qr_0^2]r_0}, & k_\psi &= \frac{2b\Xi_b[(r_0^2 + \ell^2)^2 + q\ell^2]}{\tilde{X}[(r_0^2 + \ell^2)(r_0^2 + b^2) + qr_0^2]r_0}, \\
\alpha_1 &= H_0^{-2/3} \sqrt{\frac{\sin^2 \theta \Xi_b}{\rho_0^2}} \frac{(r_0^2 + \ell^2 + q)\ell}{\ell^2 - b^2}, & \alpha_2 &= H_0^{-2/3} \sqrt{\frac{\sin^2 \theta \Xi_b}{\rho_0^2}} \frac{b(r_0^2 + b^2 + q)}{\Xi_b(b^2 - \ell^2)}, \\
\beta_1 &= H_0^{-2/3} \frac{b[(r_0^2 + \ell^2)\rho_0^2 + qr_0^2] \sin^2 \theta}{r_0 \rho_0^2 \sqrt{\ell^2 \cos^2 \theta + b^2 \sin^2 \theta} (\ell^2 - b^2)}, \\
\beta_2 &= H_0^{-2/3} \frac{\ell[(r_0^2 + b^2)\rho_0^2 + qr_0^2] \cos^2 \theta}{r_0 \Xi_b \rho_0^2 \sqrt{\ell^2 \cos^2 \theta + b^2 \sin^2 \theta} (b^2 - \ell^2)}.
\end{aligned} \tag{3.24}$$

3.3 Ultra-spinning/CFT description

In order to find the CFT dual of obtained 5d ultra-spinning black hole (3.10) as it was done through the previous section we can follow two different paths of Fig. 2. We firstly start from the lower path. In (3.22) we represented the NHEG of ultra-spinning solution. Also for the current case we choose the same boundary conditions as in [6],

$$\begin{pmatrix} \mathcal{O}(r^2) & \mathcal{O}(1) & \mathcal{O}(1) & \mathcal{O}(1/r) & \mathcal{O}(1/r^2) \\ & \mathcal{O}(1) & \mathcal{O}(1) & \mathcal{O}(1/r) & \mathcal{O}(1/r) \\ & & \mathcal{O}(1) & \mathcal{O}(1/r) & \mathcal{O}(1/r) \\ & & & \mathcal{O}(1/r) & \mathcal{O}(1/r^2) \\ & & & & \mathcal{O}(1/r^3) \end{pmatrix}, \tag{3.25}$$

using the basis $(\hat{t}, \hat{\varphi}, \hat{\psi}, \hat{\theta}, \hat{r})$. One then can show that the five-dimensional near horizon geometry (3.22) provides two copy of commuting Virasoro algebras [33], generated by a pair of commuting diffeomorphisms

$$\zeta_\varphi = -e^{in\varphi} \partial_\varphi - inr e^{-in\varphi} \partial_r, \quad \zeta_\psi = -e^{in\psi} \partial_\psi - inr e^{-in\psi} \partial_r. \tag{3.26}$$

These diffeomorphisms leading us to an asymptotic symmetry algebra of transformations that satisfy above boundary conditions. In [8] it was constructed a general form for NHEG of rotating black holes in $d = 2n + 1$ dimension

$$\begin{aligned}
ds^2 &= \Gamma(y) \left((-\rho^2 dt^2 + \frac{d\rho^2}{\rho^2}) + \sum_{\alpha=1}^{n-1} F_\alpha dy_\alpha^2 + \sum_{i,j=1}^{n-1} \tilde{g}_{ij} \tilde{e}_i \tilde{e}_j \right), \\
\tilde{e}_i &= d\phi_i + k_i \rho dt, \quad k_i = \frac{1}{2\pi T_i}.
\end{aligned} \tag{3.27}$$

The NHEG of our new solution is exactly recast in this form. The metric (3.27) has $(n - 1)$ copies of the Virasoro algebra. The central charges in this form are given by [8]

$$c_i = \frac{3}{2\pi} k_i \int d^{n-1} y_\alpha \left(\det \tilde{g}_{ij} \prod_{\alpha=1}^{n-1} F_\alpha \right)^{1/2} \int d\phi_1 \dots d\phi_{n-1}. \tag{3.28}$$

Then, for (3.22) we obtain two central charges c_1 and c_2 associated to diffeomorphisms $\partial_{\hat{\varphi}}$ and $\partial_{\hat{\psi}}$ respectively

$$\begin{aligned} c_1 &= \frac{3k_{\varphi}}{8\pi} \int \sqrt{F(\theta)(\alpha_2\beta_1 - \alpha_1\beta_2)} d\theta d\varphi d\psi = \frac{3\mu\ell[(r_0^2 + b^2)^2 + qb^2]}{\tilde{X}\Xi_b r_0^2}, \\ c_2 &= \frac{3k_{\psi}}{8\pi} \int \sqrt{F(\theta)(\alpha_2\beta_1 - \alpha_1\beta_2)} d\theta d\varphi d\psi = \frac{3\mu b[(r_0^2 + \ell^2)^2 + q\ell^2]}{\tilde{X}r_0^2}. \end{aligned} \quad (3.29)$$

We note that the central charges are containing the chemical potential μ which comes from compactification of the new azimuthal coordinate. Then similar the $U(1)^4$ case we can write the first law of thermodynamics in its extremality limit appearing as

$$TdS = -[(\Omega_{\varphi} - \Omega_{\varphi}^{ex})dJ_{\varphi} + (\Omega_{\psi} - \Omega_{\psi}^{ex})dJ_{\psi} + \sum_{i=1}^3(\Phi_i - \Phi_i^{ex}dQ_i) + (K - K^{ex})d\mu], \quad (3.30)$$

and the Boltzmann weighting factor for this case reads

$$e^{-n_R/T_R - n_{\varphi}/T_{\varphi} - n_{\psi}/T_{\psi} - \sum_{i=1}^4 Q_i/T_{t,i}}, \quad (3.31)$$

where $n_R = (E - \Omega_{\varphi}^{ex}J_{Ex} - \Omega_{\psi}^{ex}J_{\psi} - \sum \Phi_i^{ex}Q_i)r_0/\lambda$, $n_{\varphi} = J_{\varphi}$ and $n_{\psi} = J_{\psi}$. We extract again the Frolov-Thorne temperatures

$$\begin{aligned} T_R &\equiv \frac{T_H r_0}{\lambda} = 0, \\ T_{\varphi} &= -\frac{\partial T_H / \partial r_+}{\partial \Omega_{\varphi} / \partial r_+}|_{ex} = \frac{1}{2k_{\varphi}}, \quad T_{\psi} = -\frac{\partial T_H / \partial r_+}{\partial \Omega_{\psi} / \partial r_+}|_{ex} = \frac{1}{2k_{\psi}}, \end{aligned} \quad (3.32)$$

where K_{φ} and K_{ψ} are given by (3.24). Ultimately the CFT entropy using the Cardy formula is computed

$$S_{CFT} = \frac{\pi^2}{3}c_1 T_{\varphi} + \frac{\pi^2}{3}c_2 T_{\psi} = \frac{\mu\pi[(r_+^2 + \ell^2)(r_+^2 + b^2) + qr_+^2]}{4r_+\Xi_b}, \quad (3.33)$$

which is exactly same as the Bekenstein-Hawking entropy (3.16).

Now, we examine CFT duality via the upper path of Fig 2. The near horizon geometry of metric (3.3) in extremal limit was studied in [8]. One can easily check that by performing the ultra-spinning technique based on their result, our calculated NHEG (3.22) will exactly be held. It means we can again confirm that the near horizon limit commutes with the ultra-spinning limit for this 5d black hole. Hence the CFT dual of $U(1)^3$ gauged supergravity black hole (3.3) gives us the same result for both the upper and lower paths of Fig. 2.

4 Discussion

For better understanding of the physics of gauged supergravity black holes in large angular momentum, we have employed the novel ultra-spinning (super-entropic) limit proposed in [1] to generate new exact supergravity black hole solutions. In particular, we use this simple ultra-spinning technique to 4-dimensional singly spinning $U(1)^4$ and 5-dimensional doubly spinning $U(1)^3$ gauged supergravity black holes. Our obtained resulting black holes for both cases have an unusual horizon which is non-compact but with finite area. However in the first glance we see singularities in the coordinate angles

$\theta = 0, \pi$. We have shown these poles are not parts of the spacetime and can be viewed as sort of boundaries, which introduce punctures to the spacetime, providing a non-compact horizon. In [1, 13] it was shown that there is a relation between this kind of ultra-spinning limit and super-entropic black holes by exploring the thermodynamics behaviour of obtained black holes in the context of the extended phase space thermodynamics. In our upcoming work [38], we shall study the properties of these new obtained ultra-spinning black holes in the extended phase space to find the range of parameter space, giving us super-entropic black holes.

Also we have shown that the extremality conditions and the near horizon limit are preserved under ultra-spinning limit, demonstrating that they commute with the ultra-spinning limit. We have also presented the NHEG of both ultra-spinning gauged black holes despite the non-compactness of their horizons, possess an AdS_2 sector and a S^{d-2} with two punctures. Appearance of AdS_2 factor in the NHEG hints to explore, if these unusual new solutions exhibit the well-defined Kerr/CFT correspondence. We have also investigated kerr/CFT for both cases, assuming the Cardy formula we have shown that the microscopic entropy of the dual CFT for both new ultra-spinning gauged supergravity black holes in 4 and 5 dimensions is precisely agrees to its Bekenstein-Hawking entropy. Recently a formula as a higher-dimensional generalization of the Cardy formula for a large energy limit of generic CFT has been presented in [39], which relates the entropy of a CFT to the vacuum energy on $S^1 \times \mathbb{R}^d$. Now, there is a curiosity to explore, whether this formula can reproduce the entropy of the ultra-spinning black holes at a high temperature.

A further direction for our next research will be an investigation of performing hyperboloid membrane limit [15] to these supergravity black holes and their ultra-spinning versions to generate other new exact solutions.

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References

- [1] R. A. Hennigar, D. Kubiznak and R. B. Mann, Super-Entropic Black Holes, *Phys. Rev. Lett.* 115, no. 3, 031101 (2015) [arXiv:hep-th/1411.4309].
- [2] A. Strominger and C. Vafa, Microscopic origin of the Bekenstein-Hawking entropy," *Phys. Lett. B* 379, 99 (1996) [arXiv:hep-th/9601029].
- [3] A. Sen, Black Hole Entropy Function, Attractors and Precision Counting of Microstates," *Gen. Rel. Grav.* 40, 2249 (2008) [arXiv:0708.1270 [hep-th]].

- [4] H. Afshar, D. Grumiller and M. M. Sheikh-Jabbari, Near Horizon Soft Hairs as Microstates of Three Dimensional Black Holes,” [arXiv:hep-th/1607.00009].
- [5] M. M. Sheikh-Jabbari and H. Yavartanoo, Horizon Fluffs: Near Horizon Soft Hairs as Microstates of Generic AdS_3 Black Holes, [arXiv:hep-th/1608.01293].
- [6] M. Guica, T. Hartman, W. Song and A. Strominger, ”The Kerr/CFT correspondence,” [arXiv:hep-th/0809.4266].
- [7] T. Hartman, K. Murata, T. Nishioka and A. Strominger, CFT Duals for Extreme Black Holes, JHEP 0904 (2009) 019 [arXiv:0811.4393 [hep-th]].
- [8] D. Chow, M. Cvetič, H. Lu, and C. Pope, Extremal Black Hole/CFT Correspondence in (Gauged) Supergravities, [arXiv:hep-th/0812.2918v2].
- [9] J. Cardy, Operator Content Of Two-Dimensional Conformally-Invariant Theories, Nucl. Phys. B270 (1986) 186.
- [10] J. P. S. Lemos, Phys. Lett. B 353 (1995) 46 [gr-qc/9404041]; R. B. Mann, Class. Quant. Grav. 14 (1997) L109 [gr-qc/9607071]; L. Vanzo, Phys. Rev. D 56 (1997) 6475 [gr-qc/9705004]; R. -G. Cai and Y. -Z. Zhang, Phys. Rev. D 54 (1996) 4891 [gr-qc/9609065].
- [11] A. Gneccchi, K. Hristov, D. Klemm, C. Toldo, and O. Vaughan, ”Rotating black holes in 4d gauged supergravity, JHEP 1401 (2014) 127, [arXiv:hep-th/1311.1795].
- [12] D. Klemm, Four-dimensional black holes with unusual horizons, Phys.Rev. D89 (2014) 084007, [arXiv:hep-th/1401.3107].
- [13] R. A. Hennigar, D. Kubiznak, R. B. Mann and N. Musoke, Ultraspinning limits and super-entropic black holes, JHEP 1506, 096 (2015) [arXiv:hep-th/1504.07529].
- [14] R. Emparan and R. C. Myers, Instability of ultra-spinning black holes, JHEP 0309 (2003) 025, [arXiv:hep-th/0308056].
- [15] M. M. Caldarelli, R. Emparan, and M. J. Rodriguez, Black Rings in (Anti)-deSitter space, JHEP 0811 (2008) 011, [arXiv:hep-th/0806.1954].
- [16] J. Armas and N. Obers, Blackfolds in (Anti)-de Sitter Backgrounds, Phys.Rev. D83 (2011) 084039, [arXiv:hep-th/1012.5081].
- [17] M. M. Caldarelli, R. G. Leigh, A. C. Petkou, P. M. Petropoulos, V. Pozzoli, et al., ” Vorticity in holographic fluids, PoS CORFU2011 (2011) 076, [arXiv:hep-th/1206.4351].
- [18] D. Kastor, S. Ray, and J. Traschen, ”Enthalpy and the Mechanics of AdS Black Holes”, Class.Quant.Grav. 26 (2009) 195011, [arXiv:0904.2765].
- [19] M. Cvetič, G. Gibbons, D. Kubiznak, and C. Pope, ”Black Hole Enthalpy and an Entropy Inequality for the Thermodynamic Volume”, Phys.Rev. D84 (2011) 024037, [arXiv:hep-th/1012.2888].
- [20] M. Sinamuli and R. B. Mann, ”Super-Entropic Black Holes and the Kerr-CFT Correspondence”, [arXiv:hep-th/1512.07597].

- [21] Z.W. Chong, M. Cvetič, H. Lu and C.N. Pope, "Charged rotating black holes in four-dimensional gauged and ungauged supergravities", Nucl. Phys. B717, 246 (2005), [arXiv:hep-th/0411045].
- [22] M. Cvetič, G. W. Gibbons, H. Lu and C. N. Pope, "Rotating black holes in gauged supergravities: Thermodynamics, supersymmetric limits, topological solitons and time machines", hep-th/0504080.
- [23] A. Ashtekar and A. Magnon, "Asymptotically anti-de Sitter space-times", Class Quant Grav 1, L39 (1984). A. Ashtekar and S. Das, "Asymptotically anti-de Sitter space-times: Conserved quantities", Class. Quant. Grav. 17, L17 (2000), [hep-th/9911230].
- [24] W. Chen, H. Lu and C. N. Pope, Mass of rotating black holes in gauged supergravities, Phys. Rev. D 73 (2006) 104036 [hep-th/0510081]
- [25] J. M. Bardeen and G. T. Horowitz, The extreme Kerr throat geometry: A vacuum analog of $AdS_2 \times S_2$, Phys. Rev. D 60, 104030 (1999) [arXiv:hep-th/9905099].
- [26] H. K. Kunduri, J. Lucietti and H. S. Reall, Near-horizon symmetries of extremal black holes, Class. Quant. Grav. 24 (2007) 4169, [arXiv:0705.4214 [hep-th]]. H. K. Kunduri and J. Lucietti, A classification of near-horizon geometries of extremal vacuum black holes, [arXiv:0806.2051 [hep-th]]. H. K. Kunduri and J. Lucietti, Classification of near-horizon geometries of extremal black holes, Living Rev. Rel. 16 (2013) 8, [arXiv:1306.2517 [hep-th]].
- [27] K. Hajian, A. Seraj and M. M. Sheikh-Jabbari, NHEG Mechanics: Laws of Near Horizon Extremal Geometry (Thermo)Dynamics, JHEP 1403 (2014) 014 [arXiv:1310.3727 [hep-th]].
- [28] O. J. C. Dias, H. S. Reall and J. E. Santos, Kerr-CFT and gravitational perturbations, JHEP 0908 (2009) 101, [arXiv:0906.2380 [hep-th]].
- [29] M. Johnstone, M.M. Sheikh-Jabbari, J. Simon and H. Yavartanoo, Extremal Black Holes and First Law of Thermodynamics, [arXiv:1305.3157 [hep-th]].
- [30] G. Compere, "The Kerr/CFT correspondence and its extensions: a comprehensive review," Living Rev. Rel. 15, 11 (2012) [arXiv:1203.3561 [hep-th]].
- [31] G. Barnich and F. Brandt, Covariant theory of asymptotic symmetries, conservation laws and central charges, Nucl. Phys. B 633, 3 (2002) [arXiv:hep-th/0111246]. G. Barnich and G. Compere, Conserved charges and thermodynamics of the spinning Goedel black hole, Phys. Rev. Lett. 95, 031302 (2005) [arXiv:hep-th/0501102].
- [32] V.P. Frolov and K.S. Thorne, Renormalized stress-energy tensor near the horizon of a slowly evolving, rotating black hole, Phys. Rev. D39, 2125 (1989).
- [33] H. Lu, J. Mei and C. N. Pope, Kerr/CFT Correspondence in Diverse Dimensions, [arXiv:hep-th/0811.2225].
- [34] C. P. Herzog, M. Rangamani, and S. Ross, Heating up Galilean holography, JHEP 0811 (2008) 080, [arXiv:0807.1099].
- [35] M. Gunaydin, G. Sierra and P. K. Townsend, Gauging the $d = 5$ Maxwell-Einstein Supergravity Theories: More on Jordan Algebras, Nucl. Phys. B 253, 573 (1985).

- [36] S. Q. Wu, General Non-extremal Rotating Charged AdS Black Holes in Five dimensional $U(1)^3$ Gauged Supergravity: A Simple Construction Method, Phys. Lett.B707, 286 (2012) [arXiv:hep-th/1108.4159].
- [37] Z. W. Chong, M. Cvetič, H. Lu and C. N. Pope, Five-dimensional gauged supergravity black holes with independent rotation parameters, Phys.Rev. D72, 041901 (2005) [arXiv:hep-th/0505112].
- [38] Upcoming work in progress.
- [39] E. Shaghoulian, Modular forms and a generalized Cardy formula in higher dimensions, Phys. Rev.D93 no. 12, (2016) 126005, arXiv:1508.02728 [hep-th]. E. Shaghoulian, Black hole microstates in AdS, arXiv:1512.06855 [hep-th].